

# The Bivariate Brightness Function of Galaxies and a Demonstration of the Impact of Surface Brightness Selection Effects on Luminosity Function Estimations.

Nicholas Cross, Simon P. Driver

*School of Physics and Astronomy, North Haugh, St Andrews, Fife, KY16 9SS, United Kingdom*

1 February 2008

## ABSTRACT

In this paper we fit an analytic function to the Bivariate Brightness Distribution (BBD) of galaxies. It is a combination of the classical Schechter Function convolved with a Gaussian distribution in surface brightness: thus incorporating the luminosity-surface brightness correlation as seen in many recent datasets. We fit this function to a recent measurement of the BBD based on 45,000 galaxies from the *two-degree field Galaxy Redshift Survey* (Cross et al. 2001). The parameters for the best fit model are  $\phi^* = (0.0206 \pm 0.0009) h^3 \text{Mpc}^{-3}$ ,  $M_{b_j}^* - 5 \log h = (-19.72 \pm 0.04) \text{mag}$ ,  $\alpha = -1.05 \pm 0.02$ ,  $\beta_\mu = 0.281 \pm 0.007$ ,  $\mu_{e,b_j}^* = (22.45 \pm 0.01) \text{mag arcsec}^{-2}$  and  $\sigma_\mu = 0.517 \pm 0.006$ .  $\phi^*$ ,  $M_{b_j}^*$  and  $\alpha$  equate to the conventional Schechter parameters.  $\beta_\mu$  is the slope of the luminosity-surface brightness correlation,  $\mu_{e,b_j}^*$  is the characteristic effective surface brightness at  $M_{b_j}^*$  and  $\sigma_\mu$  is the width of the Gaussian.

Using a BBF we explore the impact of the limiting detection isophote on classical measures of the galaxy luminosity distribution. We demonstrate that if isophotal magnitudes are used then errors of  $\Delta M_{b_j}^* \sim 0.62 \text{ mags}$ ,  $\Delta \phi^* \sim 26\%$  and  $\Delta \alpha \sim 0.04$  are likely for  $\mu_{lim,b_j} = 24.0 \text{ mag arcsec}^{-2}$ . If Gaussian corrected magnitudes are used these change to  $\Delta M_{b_j}^* \sim 0.38 \text{ mags}$ ,  $\Delta \phi^* \sim 11\%$  and  $\Delta \alpha < 0.01$  for  $\mu_{lim,b_j} = 24.0 \text{ mag arcsec}^{-2}$ . Hence while the faint-end slope,  $\alpha$ , appears fairly robust to surface brightness issues, both the  $M^*$  and  $\phi^*$  values are highly dependent. The range over which these parameters were seen to vary is fully consistent with the scatter in the published values, reproducing the range of observed luminosity densities ( $1.1 < j_{b_j} < 2.2 \times 10^8 h L_\odot \text{Mpc}^{-3}$  see Cross et al. 2001). If total magnitudes are recovered then there is no change in the luminosity function within the errors for  $\mu_{lim,b_j} = 24.0 \text{ mag arcsec}^{-2}$ . We conclude that surface brightness selection effects are primarily responsible for this variation. After due consideration of these effects, we derive a value of  $j_{b_j} = 2.16 \times 10^8 h L_\odot \text{Mpc}^{-3}$ .

## Key words:

galaxies: luminosity function, mass function — cosmology: observations — galaxies: fundamental parameters — galaxies: general

## 1 INTRODUCTION

For the past quarter of a century the luminosity distribution of galaxies has been represented by a Schechter function (Schechter 1976). This contains three defining parameters (see Eqn. 1) and from these parameters one can derive useful quantities such as the mean local luminosity density. Over the past decade many measurements of the Schechter

parameters have been made (e.g. Efstathiou, Ellis & Peterson, 1988; Loveday et al. 1992; Marzke et al. 1994, 1998; Zucca et al., 1997). However when compared, the results from these recent surveys show a wide range in the derived Schechter parameters, Cross et al. (2001) and hence a wide range in the inferred luminosity density.

The two most likely culprits for this variation are: cosmic variance and/or surface brightness selection bias (e.g.

Disney 1976; Philipps, Davies & Disney 1990). In Cross et al. (2001) these two errors were quantified for a sample of 45,000 galaxies drawn from the two-degree field galaxy redshift survey (2dFGRS). The conclusion was that both of these issues are significant and can potentially lead to significant underestimates of the local luminosity density ( $\sim$ few % for cosmic variance and  $\sim$ 35% for surface brightness biases). The surface brightness biases can be separated into two distinct parts; a surface brightness dependent photometric bias and a surface brightness dependent Malmquist bias. The biggest effect comes from the former, i.e. underestimating the total magnitudes (25%). The remaining contribution comes from overestimating the volume over which a galaxy can be seen (10%). Cross et al. (2001) advocated the construction of a Bivariate Brightness Distribution (BBD) to remove the surface brightness selection biases. The BBD is the space density of galaxies as a function of the absolute magnitude,  $M$ , AND the absolute surface brightness,  $\mu$ . This process also revealed a number of noteworthy results in its own right: Firstly a clear dearth of giant low-surface brightness galaxies; secondly a clear luminosity-surface brightness correlation (spanning  $-23 < M_{b_j} < -16$ ); and thirdly the general rise in the space-density from giant to dwarf systems to the faint limit ( $M_{b_j} = -16$ ).

Some earlier attempts have been made to measure the BBD (e.g. Choloniewski 1985; van der Kruit 1987; Sodr  & Lahav 1993). These were for small samples of bright galaxies. The Choloniewski sample contained 248 E/S0 galaxies with  $-22 < M_B < -18$ ; the van der Kruit sample contained 51 galaxies with isophotal diameters  $> 2'$  at 26.5 mag arcsec $^{-2}$  (in the photographic IIIa-J band) and the Sodr  & Lahav sample contained 529 galaxies with isophotal diameters  $> 1'$  at 25.6-26.0 mag arcsec $^{-2}$  in the B-band. These are very strict limits, and only very intrinsically bright and large galaxies were well sampled.

Recently three more extensive and independent measurements of the Bivariate Brightness Distribution have been made. Driver (1999) derived the BBD for a volume-limited sample drawn from the Hubble Deep Field, de Jong & Lacey (2000) studied a homogeneous sample of 1000 late-type spirals and Blanton et al. (2001) derived the BBD for a sample of 11,275 galaxies from the Sloan Digital Sky Survey. These three surveys plus Cross et al. (2001) all confirm the existence of the luminosity-surface brightness correlation, demonstrating that *surface brightness selection biases are luminosity dependent*. The luminosity-surface brightness correlation is measured to be  $M_{b_j} = 2.4 \pm_{0.5}^{1.5} \mu_e - 72.3 \pm_{32.9}^{11.0}$  from (Cross et al. 2001). This equates to  $\mu_e \propto 0.42 \pm_{0.26}^{0.10} M_{b_j} + 30.2 \pm_{5.1}^{2.1}$ . From Hubble Deep Field data, Driver (1999) found a steeper gradient ( $M_{F450W} \propto 1.5 \mu_e$ ) as did Ferguson & Binggeli (1994) in the Virgo cluster ( $M_B \propto 1.4 \mu_o$ ). The number density of galaxies is a maximum along this line, falling away at both higher and lower surface brightnesses. Given the luminosity-surface correlation it is hardly surprising if surveys with differing selection criterion recover widely ranging Schechter parameters.

A useful next step is to produce an analytical function to fit to these derived BBDs. In §2 we show such a function and fit it to the derived 2dFGRS BBD. In §3 we discuss our fitting procedure and compare it to a similar estimation made by de Jong & Lacey (2000). In §4 we show how the luminosity density can be calculated from the BBF and

compare it to the published values. Finally we explore the scope for error in Schechter function estimators if surface brightness selection effects are ignored.

## 2 A BIVARIATE BRIGHTNESS FUNCTION

The luminosity function as traditionally described by the Schechter Function (Schechter 1976), is shown below as Eqn. 1:

$$\phi(M)dM = 0.4 \ln(10) \phi^* 10^{0.4(M^* - M)(\alpha + 1)} e^{-10^{0.4(M^* - M)}} \quad (1)$$

This equation contains three parameters  $M^*$ ,  $\phi^*$  &  $\alpha$  which describe the ‘‘characteristic magnitude’’, the ‘‘normalisation constant’’ and the ‘‘faint end slope’’, respectively. Containing no surface brightness information this function provides a good fit to the space density of field galaxies albeit over a fairly restricted range of luminosities ( $-22 < M_{b_j} < -16$ ) in the field. However, the Schechter Function provides a poorer fit to the luminosity distribution in clusters (e.g. Driver et al. 1994, Andreon, Cuillandre & Pell  2000) and when the population is subdivided according to spectral type (Madgwick et al. 2001). Current estimates constrain the three Schechter parameters, for field galaxies, to lie in the range:  $-19.75 < M^* < -19.15$ ;  $0.013 < \phi^* < 0.027$ ;  $-1.22 < \alpha < -0.7$  resulting in a pessimistic luminosity density range of:  $1.1 < j_{b_j} < 3.2 \times 10^8 h L_\odot \text{Mpc}^{-3}$ .

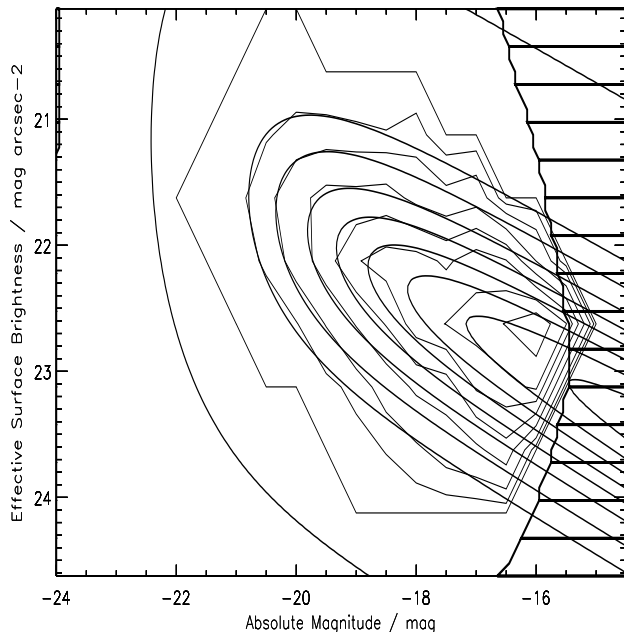
Empirically Cross et al. (2001) find that the distribution of the galaxy population in surface brightness appears symmetrical about a ridge and can therefore be described by a Gaussian or possibly a quadratic distribution. The ridge is described by  $\mu = \beta_\mu M + C$ , the luminosity-surface brightness correlation. Clearly a BBF needs to relate closely to the Schechter function in luminosity. Hence by multiplying the classical Schechter function with a Gaussian in surface brightness we can construct the following BBF:

$$\phi(M, \mu_e) = \frac{0.4 \ln(10)}{\sqrt{2\pi} \sigma_\mu} \phi^* 10^{0.4(M^* - M)(\alpha + 1)} e^{-10^{0.4(M^* - M)}} \exp\left[-\frac{1}{2} \left( \frac{\mu_e - \mu_e^* - \beta_\mu (M - M^*)}{\sigma_\mu} \right)^2\right] \quad (2)$$

where  $\beta_\mu$  is the gradient of the luminosity-surface brightness correlation and  $\sigma_\mu$  is the dispersion in the surface brightness. This function is identical to that presented by Choloniewski (1985), and derived by Dalcanton, Spergel & Summers (1997) and de Jong & Lacey (1999a,b 2000) using the Fall & Efstathiou (1980) disk-galaxy formation model. Note that the new term contains a normalisation coefficient  $\frac{1}{\sqrt{2\pi} \sigma_\mu}$  ensuring that  $\phi^*$ ,  $M^*$  and  $\alpha$  are identical to the traditional Schechter parameters.

## 3 FITTING THE BBF

We choose to fit the BBF to the data shown in Cross et al. (2001); this is the largest available data set. This BBD was derived from a subset of 45,000 galaxies from the two-degree field galaxy redshift survey (see Cross et al. 2001 for



**Figure 1.** The thick lines show the BBF computed for the best fit parameters. The thin lines depict the 2dFGRS BBD (q.v. Cross et al. 2001 Fig. 10b). The contours are at  $1.0 \times 10^{-7}$ ,  $1.0 \times 10^{-3}$ ,  $2.5 \times 10^{-3}$ ,  $5.0 \times 10^{-3}$ ,  $7.5 \times 10^{-3}$ ,  $1.0 \times 10^{-2}$ ,  $1.25 \times 10^{-2}$ ,  $1.5 \times 10^{-3}$ ,  $1.75 \times 10^{-2}$ ,  $2.0 \times 10^{-3}$  galaxies  $\text{Mpc}^{-3} \text{mag}^{-1} (\text{mag arcsec}^{-2})$ . The shaded area shows the selection boundary - see Cross et al. (2001) for details.

details), and is shown on Fig. 1 (thin contours). Also shown in Fig. 1 is the selection boundary derived from visibility theory (shaded region, see Appendix B of Cross et al. 2001). In the shaded region insufficient volume is surveyed to make any meaningful statement of the space-densities.

The BBF can be fitted to the BBD by minimising the  $\chi^2$  of the model compared to the data. The BBF is a non-linear six parameter equation and to find the minimum, we use the Levenberg-Marquardt Method (see Press et al. 1986). The data provided (see Cross et al. 2001) is binned as in Table C2 of Cross et al. (2001) using those bins whose values are binned on a minimum of 25 galaxies.

The best fit parameters we derive are:  $\phi^* = (0.0206 \pm 0.0009)h^3 \text{Mpc}^{-3}$ ,  $M^* - 5 \log h = (-19.72 \pm 0.04) \text{mag}$ ,  $\alpha = -1.05 \pm 0.02$ ,  $\beta_\mu = 0.281 \pm 0.007$ ,  $\mu_e^* = (21.90 \pm 0.01) \text{mag arcsec}^{-2}$  and  $\sigma_\mu = 0.517 \pm 0.006$ . All errors are  $1\sigma$  errors. Fig. 1 shows the BBF for these parameters (thick lines) overlaid on the data (thin contour lines). The errors in the parameters were found using a Monte-Carlo simulation, that is the observed distribution was randomised within the quoted  $1\sigma$  errors and the BBF fit re-derived. The final BBF fit yields a  $\chi^2$  value of 164, for  $\nu = 49$ , where  $\nu$  is the no. of data points - no. of parameters. This gives a likelihood of  $2.6 \times 10^{-14}$ .

Hence, although the BBF appears to describe the BBD, the fit is poor. It is important to understand where the differences are occurring. From Fig. 1 we see that the model fits the data well brightwards of  $M = -18$ , and less well in fainter bins. The errors become comparable to the space density faintwards of  $M = -16$ , so the main error in the fit

occurs in the range:  $-16 > M > -18$ . The data (Fig. 1) show an upturn towards the faint end in this range whereas the Schechter function gradually flattens towards the faint end. Thus it is the Schechter function part that does not describe the data well. Note that the BBF provides Schechter parameters comparable to the range from previous surveys.

The model fits the data well in the surface brightness direction, implying that a Gaussian distribution is a good description of the space density as a function of surface brightness, for a constant absolute magnitude.

### 3.1 Comparison with de Jong & Lacey and Blanton

Table 1 compares our BBF with the de Jong & Lacey (2000) BBF which was determined for Sb-Sdm galaxies only. As de Jong & Lacey use 95% confidence intervals for their errors, we have quoted  $2\sigma$  errors rather than  $1\sigma$  errors for our values. We converted their half-light radii parameters to our effective surface brightness parameters. De Jong & Lacey fit disks and exponential bulges to their data, taking into account inclination and internal extinction. In Appendix A, we estimate the uncertainty in our results due to not taking this into account and find that the error is  $\sim 0.1 \text{mag}$  in  $M^*$  and  $\sim 0.55 \text{mag arcsec}^{-2}$  in  $\mu_e^*$ . We note that the de Jong & Lacey (2000) parameters are their total galaxy parameters, not their disk-only galaxy parameters. In each case, these have been converted to  $b_j$ -band magnitudes using  $B - I = 1.7 \text{mag}$  from de Jong & Lacey (2000), and  $b_j = B - 0.28(B - V)$  (Maddox, Efstathiou & Sutherland 1990), using a value of  $(B - V) = 0.5$  for a late type spiral (Coleman, Wu & Weedman, 1980, Driver et al. 1994). Finally we convert from  $H_0 = 65 \text{km s}^{-1} \text{Mpc}^{-1}$  to  $H_0 = 100 \text{km s}^{-1} \text{Mpc}^{-1}$ . In addition, the de Jong & Lacey sample has tighter selection criteria and more accurate CCD photometry ( $\pm 0.05 \text{mag}$  compared to  $\pm 0.2 \text{mag}$ ), but it only includes late-type galaxies and has a redshift completeness of 80% compared to  $> 90\%$  for the 2dFGRS. In spite of this, we find a similar spread in  $\mu$  ( $\sigma_\mu = 0.52$  q.v. 0.61) and we find that the  $\alpha$  values of the two surveys are equal within the errors. The 2dFGRS has a brighter  $M^*$  by 0.05 mag, and a brighter  $\mu_e^*$  by  $0.85 \text{mag arcsec}^{-2}$ . Taking into account the effects of bulges and inclination as mentioned above, the 2dFGRS distribution has become 0.15 mag brighter than the de Jong & Lacey distribution and has a brighter  $\mu_e^*$  by 0.3 mag  $\text{arcsec}^{-2}$ . Considering that late-type galaxies tend to be fainter and lower surface brightness, these results appear fully consistent.

Our value of  $\beta_\mu$  can be converted to a luminosity-scale size gradient  $\beta_{r_e} = -0.360 \pm 0.004$ . Although this differs from the de Jong & Lacey (2000) value, interestingly, it agrees more closely with their theoretical prediction of  $\beta_{r_e} = -\frac{1}{3}$  (see de Jong & Lacey 2000). One possible reason for the variation in  $\beta_\mu$  may be a correlation between colour and absolute magnitude. Blanton et al. (2001) find a strong correlation between  $(g^* - r^*)$  colour and  $M_{r^*}$ : brighter galaxies are redder, fainter galaxies are bluer. Making estimates of the colour-magnitude correlation from Fig. 13 of Blanton et al. (2001), we find that  $M_{r^*} = -8.75 \pm \frac{4}{2}(g^* - r^*) - 14.88 \pm \frac{1}{2}$ . Using a mean  $b_j - r^* = 1.1$  (calculated from Fukugita, Shimasaku & Ichikawa 1995 and Maddox, Efstathiou & Sutherland 1990), and assuming that the additional colour

term  $\Delta(b_j - r^*) = \Delta(g^* - r^*)$  we calculate that the expected  $\beta_{\mu, r^*} = 0.36 \pm 0.23$ . The value estimated from Fig. 10 of Blanton et al. (2001) is  $\beta_{\mu, r^*} = 0.50 \pm 0.1^2$ . Thus the  $r^*$  band luminosity-surface brightness correlation appears steeper than the  $b_j$  band luminosity-surface brightness correlation. A similar colour-magnitude correlation in  $(b_j - I)$  could explain the discrepancy between our result and de Jong & Lacey (2000) result.

The general good overall agreement between these substantially different surveys is an important vindication of both results. Cross et al. (2001) has extended the de Jong & Lacey conclusions to the full range of galaxy types with  $M < -16$ . However, the different values obtained for the luminosity surface brightness correlation may reflect a colour or morphologically dependent luminosity-surface brightness correlation. Blanton et al. (2001) seem to have found similar results, but have not fitted a function or tabulated their results.

#### 4 CALCULATING THE LUMINOSITY DENSITY

As for the Schechter function it is trivial to calculate the luminosity density,  $j$ , by integrating the product of the BBF and the luminosity over the complete range of surface brightness and absolute magnitude.

$$j = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L(M) \phi(M, \mu) dM d\mu \\ = \phi^* L^* \Gamma(\alpha + 2) = \phi^* 10^{-0.4(M^* - M_{\odot})} \Gamma(\alpha + 2) \quad (3)$$

The solution is the same as the solution to the integral obtained from the Schechter function. When calculated using the best fit parameters, the value of the luminosity density,  $j_{b_j} = (2.16 \pm 0.14) \times 10^8 h L_{\odot} \text{Mpc}^{-3}$ \*

In Blanton et al. (2001), the Sloan team get a 40% higher value for the luminosity density in the  $b_j$  filter than the 2dFGRS team. Does this mean that 2dFGRS is missing some galaxies, or at least underestimating their fluxes? For a start the values of  $M^*$  are consistent, suggesting that both surveys are correcting magnitudes properly. However the measurement of  $\phi^*$  is over 30% higher in Blanton et al. (2001). A more recent paper (Yasuda et al. 2001) revises the SDSS luminosity density of Blanton et al., to  $j_{b_j} = 2.43 \pm 0.21 \times 10^{-2} h^3 \text{Mpc}^{-3}$ . This revision is based on a fit to the galaxy number counts, suggesting that the Blanton et al. region was overdense by 30%. The revised  $\phi^*$  value ( $\phi^* = 2.05 \pm 0.12_{-0.28}^{+0.66} \times 10^{-2} h^3 \text{Mpc}^{-3}$ ) is now consistent with our measurement of  $\phi^* = 2.06 \pm 0.09 \times 10^{-2} h^3 \text{Mpc}^{-3}$ , in the  $b_j$  band.

Given this revised value of  $\phi^*$ , the Blanton result is still 10% higher, but Blanton used a colour term  $b_j = B - 0.35(B - V)$ , whereas the correct colour term for the APM, tested using EIS data is  $b_j = B - 0.28(B - V)$  (Peacock, private communication). When these two factors are taken into account, the luminosity densities are entirely consistent.

\* Cross et al. (2001) used a value of  $L_{\odot, b_j} = 5.48$ , rather than the correct value of  $L_{\odot, b_j} = 5.30$ . The values for the luminosity density in this paper use the correct value.

As demonstrated in Cross et al. (2001) the peak of the luminosity density lies well inside the selection boundaries. When the function is integrated over the range  $-24 < M < -15.5$ ,  $20.1 < \mu_e < 24.1$ , the value obtained is  $j_{b_j} = 2.14 \times 10^8 h L_{\odot} \text{Mpc}^{-3}$  as compared to the summed BBD which gives:  $j_{b_j} = (2.11 \pm 0.20) \times 10^8 h L_{\odot} \text{Mpc}^{-3}$ . These are the approximate selection boundaries, so the correspondence is excellent. Unless the distribution shows an upturn outside the selection boundary the 2dFGRS data have uncovered over 98% of the local B-band luminosity density.

#### 5 EXPLORING SURFACE BRIGHTNESS SELECTION EFFECTS

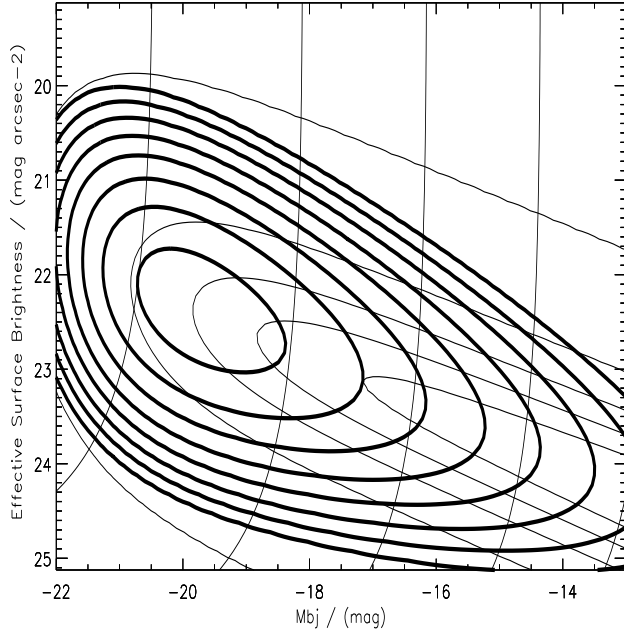
In Cross et al. (2001), Fig. 1 showed the variation in the LF as measured from a number of recent redshift surveys. It was postulated that the large variation was due to surface brightness selection effects. In this section the variation of the luminosity function with the limiting detection isophote is explored and compared to the range of published luminosity functions. Throughout, an exponential surface brightness profile and an Einstein-de Sitter cosmology are assumed.

To calculate a derived luminosity function, we start with our fit to the BBF. We take into account the overestimate in  $\mu_e^*$  by  $0.55 \text{ mag arcsec}^{-2}$ , derived in Appendix A, and use a value of  $\mu_e^* = 22.45 \text{ mag arcsec}^{-2}$  [Note that in Cross et al. (2001) we used a less sophisticated method to estimate the offset caused by the bulge and arrived at a figure of  $0.55 \text{ mag arcsec}^{-2}$ ].

We multiply our updated BBF by a visibility volume (Cross et al. 2001) to construct an apparent observed number in  $M$  and  $\mu$  (see Fig. 2). The parameters adopted to derive the visibility surface are:  $m_{faint} = 20.0 \text{ mag}$ ,  $m_{bright} = 14.0 \text{ mag}$ ,  $d_{min} = 2.0''$ ,  $d_{max} = 250.0''$ ,  $z_{max} = 0.5$ ,  $z_{min} = 0$ . The solid angle used was  $300 \square^\circ$ . These parameters are typical of the observed ranges for the most recent surveys. The only parameter allowed to vary is the detection isophote which took the values 26, 25, 24, 23.5 and 23  $\text{mag arcsec}^{-2}$ . Fig. 3 shows the complex Malmquist bias for  $\mu_{lim} = \infty, 26, 24, 23 \text{ mag arcsec}^{-2}$  (top to bottom). The shaded region shows the approximate location of the galaxy population. The lines are contours of constant volume for that  $\mu_{lim}$ . As the limiting isophote becomes brighter, the surface brightness dependency of the Malmquist bias increases.

Given the observed distribution, each galaxy within each bin is then randomly assigned a volume out to the maximum derived from visibility theory. This volume is converted to a redshift assuming an Einstein-de Sitter cosmology and a standard k-correction. We assume that the number density does not vary within the bin as a function of redshift, i.e. there is no evolution and no clustering. The exact absolute magnitude and surface brightness value is then randomly assigned within each bin, (this assumes that the number density does not vary within the bin as a function of  $M$  or  $\mu_e$ ). Around the  $M^*$  point particularly, this assumption fails, but a Monte Carlo simulation done at higher resolution finds no significant differences. The numbers in Table 2, and the plots in in Fig 2- 5 were produced from this Monte Carlo simulation.

The net result is a magnitude-limited sample with ob-

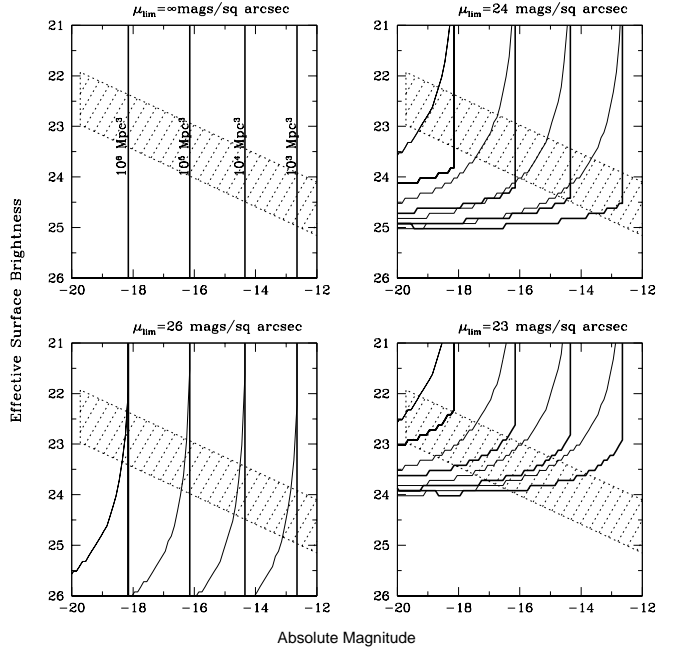


**Figure 2.** This figure shows 3 contour plots. The thin lines show the contours of the visibility surface, in  $\text{Mpc}^3 \text{bin}^{-1}$ . Each line is a decade apart. The five lines shown range from  $10^7$  to  $10^3$ , from the bright end to the faint end respectively. This visibility surface has a detection threshold of  $26.0 \text{ mag arcsec}^{-2}$  and the limits are in isophotal magnitudes (see §5). The medium thickness lines show the BBF from Fig. 1, offset in surface brightness by  $0.55 \text{ mag arcsec}^{-2}$ . The thick lines show the number of galaxies detected in each bin. The contours levels are at 1.0, 3.2, 10, 32, 100, 320, 1000, 3200 and 10000 galaxies  $\text{mag}^{-1} (\text{mag arcsec}^{-2})^{-1}$ .

jects randomly distributed within their allowed volume. We now calculate each galaxy's isophotal magnitude, a Gaussian<sup>†</sup> corrected magnitude and their total magnitude and sum the final number distribution according to absolute magnitude. This is plotted in Fig 4 for total, corrected and isophotal absolute magnitudes.

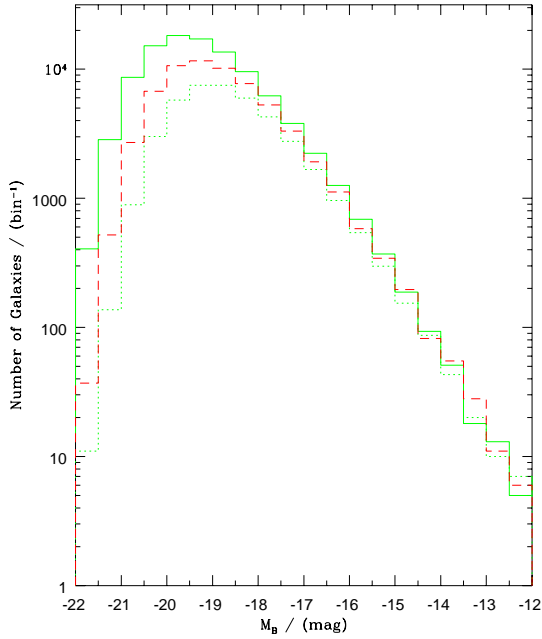
We then reconstruct the luminosity function using a  $1/V_{\text{Max}}$  prescription (as our simulations contain no clustering this should be an optimal estimator). Fig. 5 shows the recovered luminosity functions. The LFs of Fig. 5 demonstrate the impact of surface brightness selection as they are all drawn from the same BBF; the only difference is the limiting isophote and the choice of magnitude measurement. The range of published values is shown as the shaded area

<sup>†</sup> One simple and popular method to correct for light lost is the Gaussian correction employed by Maddox, Efstathiou & Sutherland (1990) on the APM, and used as a correction in the Source Extractor code (Bertin & Arnouts 1996). It works by fitting a Gaussian with central surface brightness,  $\mu_0$ , and standard deviation,  $\sigma$ , to the light profile of the galaxy, such that the isophotal radius of the Gaussian matches the isophotal radius of the galaxy and the isophotal magnitude of the Gaussian is equivalent to the isophotal magnitude of the galaxy. The Gaussian corrected magnitude is then the total flux under the Gaussian. This works well for compact objects such as stars and small angular scale size galaxies where the seeing dominates the profile.



**Figure 3.** This plot shows contours of equal volume for different selection functions. The top, left plot shows the selection function for an infinite threshold. The selection lines have no surface brightness dependence. The other plots show isophotal (thin lines) and total (thick lines) selection functions, for the detection thresholds 26, 24, & 23  $\text{mag arcsec}^{-2}$ . As one goes to brighter thresholds, the volume becomes a stronger function of surface brightness. Thus the mean volume at any  $M$  decreases, and a volume correction or estimator which is only magnitude dependent becomes more biased. The shaded parallelograms represent the number density distribution, in terms of the luminosity-surface brightness correlation, and the width of the surface brightness distribution. Where the contour lines are vertical, the volume with magnitude dependence only is unbiased, elsewhere it is biased. Where the contour line crosses the distribution, the luminosity function can be recovered using an estimator which takes into account surface brightness. Where the distribution is missed, there is input catalogue incompleteness, and so there is not enough information to recover the luminosity function. When total magnitudes are used: at  $26 \text{ mag arcsec}^{-2}$ , there is no surface brightness dependency within the shaded region for  $M < -12$ , so a magnitude-only will give an unbiased luminosity function; at  $24 \text{ mag arcsec}^{-2}$ , it will be unbiased for  $M < -14.3$ , but an estimator with surface brightness built in, such as the method in Cross et al. (2001), will be unbiased  $M < -12$ ; at  $23 \text{ mag arcsec}^{-2}$ , the magnitude only estimator will be biased at all magnitudes, but a surface brightness estimator will recover the luminosity function for  $M < -16$ . Using isophotal magnitudes, all luminosity function estimators will be biased if they are only magnitude dependent.

(excluding the LCRS Lin et al. 1996). Also shown is the limit solution, for our model BBF. The left panel assumes isophotal magnitudes are measured, the central panel assumes Gaussian corrected magnitudes were used and the right panel assumes some procedure has been implemented to recover the total magnitudes. The results are also tabulated in Table 2.



**Figure 4.** This is a plot of the number of galaxies detected as a function of absolute magnitude, for a limiting isophote of 24.0 mag arcsec<sup>-2</sup>. The solid line shows the plot for total magnitude, the dashed line shows the corresponding corrected magnitudes and the dotted line shows the corresponding isophotal magnitude. The difference between each line is a combination of an offset in magnitude and fewer galaxies being detected going from total to isophotal magnitudes. The peak of the total magnitudes is  $\sim 0.3$  mag brighter than the peak in the corrected magnitudes which is  $\sim 0.35$  mag brighter than the peak in the isophotal magnitudes.

### Isophotal magnitudes

If isophotal magnitudes are adopted and the surface brightness limit is bright, the luminosities of galaxies are severely underestimated. Thus both the number density and the  $M^*$  value are severely underestimated (see Table 2). The variation in  $\phi^*$  is up to 50% and in  $M^*$  up to 1.0 mags. This tallies well with the range of Schechter values recovered (see §2) over the range tested ( $23 < \mu_{lim} < 26$ ). To some extent it is surprising that  $\phi^*$  is not more drastically effected; this is because the observed distribution of galaxies is skewed towards the faint end, see Fig. 4. As a simple  $1/V_{max}$  correction or maximum likelihood estimator based on the isophotal magnitudes alone does not take into account surface brightness issues, especially light loss, a smaller volume is calculated than for total magnitudes, leading to an overestimate of the number density, see Fig. 3. This is tempered by a lower number density at brighter absolute magnitudes.

Perhaps most surprising is the robustness of the faint end slope whose value is recovered correctly regardless of the isophote.

### Corrected magnitudes

Most surveys attempt to correct their isophotal magnitudes to total magnitudes. We used a Gaussian correction as described above. Fig. 5 and Table 2 demonstrate that corrected

magnitudes recover 63% of the luminosity density at 24 mag arcsec<sup>-2</sup> compared to the 43% that isophotal magnitudes recover and the 98% that total magnitudes recover. As with isophotal magnitudes, corrected magnitudes give a luminosity function biased at all values of  $M$ , although the bias has been significantly reduced.

### Total magnitudes

If some method is employed to correct the galaxies to total magnitudes (e.g. Kron magnitudes or Petrosian magnitudes) we find that the parameters are very robust for  $\mu_{lim} \geq 24$  mag arcsec<sup>-2</sup>. However, at  $\mu_{lim} = 23$  mag arcsec<sup>-2</sup> the number density is underestimated throughout the distribution. Fig 3 illustrates why this occurs.

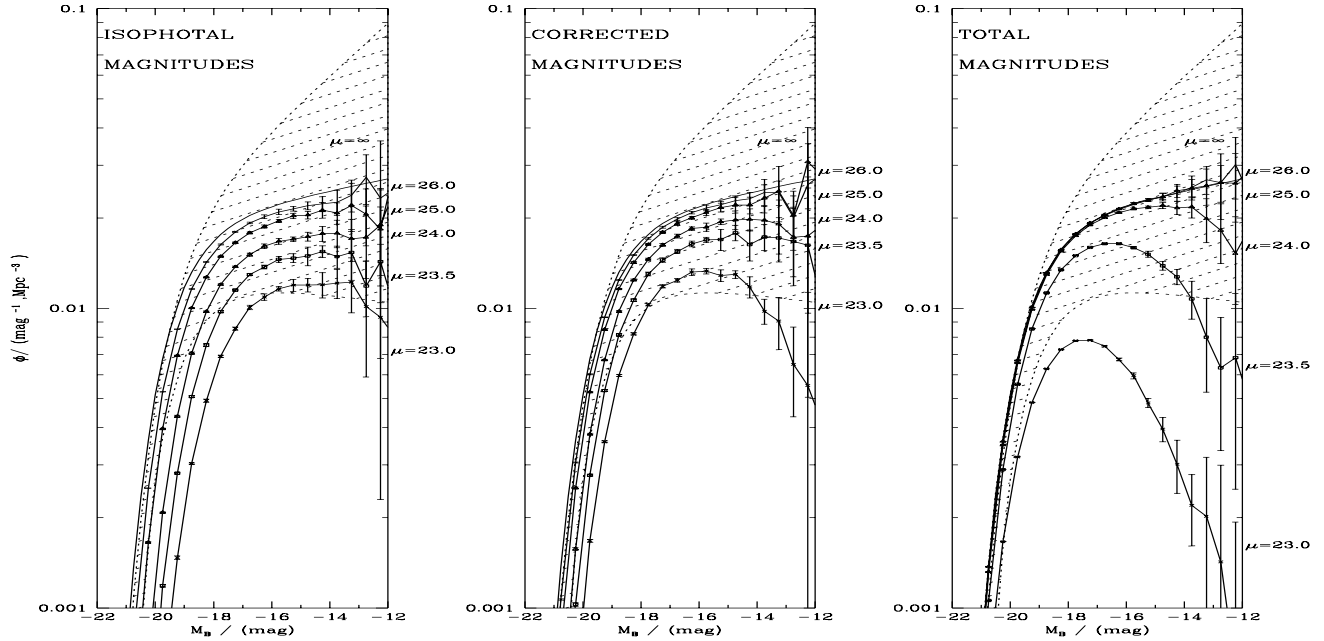
The volume has almost no surface brightness dependency for the thresholds  $24 < \mu_{lim} < 26$  provided  $M < -14$ , but it has significant surface brightness dependency for  $-22 < M < -14$  at  $\mu_{lim} = 23$ . The bright absolute magnitudes are affected because of cosmological dimming and the K-correction. Galaxies at the maximum redshift of  $z = 0.5$  will have an apparent surface brightness 3 mag arcsec<sup>-2</sup> fainter than their intrinsic surface brightness (1.7 mag arcsec<sup>-2</sup> due to cosmological dimming and 1.3 mag arcsec<sup>-2</sup> due to the K-correction, where  $K(z) = 2.5z$ ). Even galaxies at  $z = 0.25$  will be fainter by 1.6 mag arcsec<sup>-2</sup>. Thus a galaxy with central surface brightness of 21.5 mag arcsec<sup>-2</sup> ( $\mu_e = 22.6$  mag arcsec<sup>-2</sup>) and  $z = 0.25$  will not be detected with a threshold of 23 mag arcsec<sup>-2</sup>.

Recovering total magnitudes beforehand will give good estimators for the luminosity function, provided that significant numbers of galaxies are not missing. This is a particular problem if your maximum redshift is very high. However, if galaxies are missing because of cosmological effects, rather than being too intrinsically dim even at  $z = 0$ , the number density can be recovered using a surface brightness dependent volume correction as discussed in Cross et al. (2001).

### Overall Effects

Overall the variations recovered in  $M^*$ ,  $\phi^*$  and  $\alpha$  between simulated surveys with limits of  $24 < \mu < 26$  (i.e., comparable to existing surveys) are  $-19.74 < M^* < -19.08$ ,  $0.020 < \phi^* < 0.017$  and  $-1.07 < \alpha < -1.05$ . Fig. 5 shows that the observed variation in Schechter function parameters has been recovered at the bright end for  $24 < \mu_{lim} < 26$  when either isophotal or Gaussian corrected magnitudes are used. However, Fig. 5 demonstrates that the observed variation in the faint end slope has not been recovered and is testament to the fact that surface brightness selection effects *do not* reproduce all the variation seen in the faint-end slope. This supports the suggestion in Cross et al. (2001) that the faint-end slope depends more critically on the clustering correction than surface brightness issues.

Blanton et al. (2001) find a 0.08 change in the faint end slope going from  $\mu_{e,r^*} = 23.5$  to  $\mu_{e,r^*} = 24.5$ , with no change at the bright end. Using a  $(b_j - r^*) = 1.1$ , this leads to a  $b_j$ -band isophote of  $\mu_{lim} = 23.5$  for  $\mu_{e,r^*} = 23.5$ . The change in the faint end can be compared to the changes seen in Table 2, for total magnitudes. The faint end slope,  $\alpha \sim -1.037$  for  $\mu_{lim} = 23.5$  and  $\alpha \sim -1.058$  for  $\mu_{lim} = 24.5$ .



**Figure 5.** The variation of LFs with  $\mu_{lim}$  for isophotal (top), corrected (middle) and total (bottom) magnitudes.  $\mu_{lim}$  varies from 26 mag arcsec $^{-2}$  to 23 mag arcsec $^{-2}$ . The shaded region shows the variation of recent surveys from Sloan (with the new  $\phi^*$ ) to APM. The LCRS is not shown as it has an additional surface brightness constraint that significantly reduces the faint end.

This gives a 0.021 change over a similar interval, lower than the Sloan result. However, Sloan has a red selected sample, which gives a steeper luminosity-surface brightness correlation (see §3.1). This could account for a greater change in  $\alpha$ .

In Cross et al. (2001) we take an isophotally selected sample and apply corrections in surface brightness as well as absolute magnitude. These corrections imply that we will not underestimate  $M^*$  or  $\phi^*$ . SDSS calculated Petrosian magnitudes before selecting their sample. Petrosian magnitudes are aperture magnitudes and therefore do not show such a pronounced variation with redshift as isophotal magnitudes. Thus the sample is selected from pseudo-total limits.

For deep isophotes, the volume correction has virtually no surface brightness dependence, as shown in Fig 5, so the number density can be calculated at the bright end trivially and is only underestimated at the faint end where some galaxies have too a low surface brightness to get into the sample. Using either of the techniques outlined in the previous paragraph (Cross et al. 2001, Blanton et al. 2001) should give accurate values of  $M_b^* - 5 \log h$  and  $\phi^*$ ,  $-19.75 \pm 0.05$  mag,  $(2.02 \pm 0.02) \times 10^{-2} h^3 \text{Mpc}^{-3}$  for 2dFGRS (Cross et al. 2001) and  $-19.70 \pm 0.04$  mag,  $(2.05 \pm 0.12) \times 10^{-2} h^3 \text{Mpc}^{-3}$  for SDSS (Blanton et al. 2001, Yasuda et al. 2001). The caveat is that no correction to total magnitudes is perfect, and the corrected magnitudes will tend to have some surface brightness dependency as is demonstrated by the Gaussian corrected luminosity function. Even when isophotal magnitudes have been corrected to pseudo-total magnitudes it is

better to use a  $1/V_{max}$  or maximum likelihood estimator which is a function of both  $M$  and  $\mu$ .

While a good understanding of visibility theory will account for galaxies within the surface brightness limits, galaxies with very low central surface brightness, but bright total magnitudes can be missed. These are one source of mismatches between the estimates of  $\alpha$  in different surveys. Another reason for different estimates is inhomogeneities in the space density of galaxies. Surveys looking at different parts of the sky will encounter variations in the Large Scale Structure. Dwarf galaxies are seen over a smaller volume, so they can have large clustering corrections. Differences in clustering corrections between different surveys will tend to bias  $\alpha$  rather than  $\phi^*$ .

## 6 CONCLUSIONS

We have presented a fitting function for the Bivariate Brightness Distribution. This takes a similar form to a Schechter function in luminosity coupled with a Gaussian distribution in surface brightness. The Bivariate Brightness Function was fitted to the recent results from Cross et al. (2001) who constructed a BBD for a sample of 45,000 galaxies from the 2dFGRS. The BBF fits the data well at the bright end, but poorly at the faint end.

We compare the parameters of the fit to the de Jong & Lacey (2000) results for late-type spirals. While our results broadly agree there are differences which may provide clues toward understanding formation and evolution tracks of different galaxy types.

The BBF can be integrated to yield a total luminosity density of  $j_{b_j} = 2.16 \pm 0.14 \times 10^8 h L_\odot \text{Mpc}^{-3}$ . This agrees with the luminosity density calculated from the data and demonstrates that unless the BBD shows sub-structure or a dramatic upturn beyond the selection boundaries the majority of the luminosity density in the local Universe has now been detected.

This paper has dealt with the biases that occur if you start with an isophotally selected sample and do not apply any corrections, or apply a light loss correction without properly considering the volume correction. In both cases the Schechter parameters can be biased, and the luminosity density underestimated.

Using a BBF we explore the impact of the limiting detection isophote on classical measures of the Schechter function. We demonstrate that if isophotal magnitudes are used then errors of  $\Delta M_{b_j}^* \sim 0.62$  mags,  $\Delta \phi^* \sim 26\%$  and  $\Delta \alpha \sim 0.04$  are likely at  $\mu_{lim,b_j} = 24.0 \text{ mag arcsec}^{-2}$ . If Gaussian corrected magnitudes are used these change to  $\Delta M_{b_j}^* \sim 0.38$  mags,  $\Delta \phi^* \sim 11\%$  and  $\Delta \alpha < 0.01$  are likely at  $\mu_{lim,b_j} = 24.0 \text{ mag arcsec}^{-2}$ . If total magnitudes can be recovered then the observed luminosity function will be correct within the errors provided  $\mu_{lim,b_j} > 24.0 \text{ mag arcsec}^{-2}$ . Hence while the faint-end slope,  $\alpha$ , appears fairly robust to surface brightness issues both the  $M^*$  point and  $\phi^*$  are highly dependent. The range over which these parameters vary is fully consistent with the scatter in the published values which come from a variety of surveys with differing selection criterion. These parameters produce a range in the luminosity density,  $j_{b_j}$ , of  $0.9 < j_{b_j} < 2.2 \times 10^8 h L_\odot \text{Mpc}^{-3}$  again agreeing well with the range of published values ( $1.1 - 2.2 \times 10^8 h L_\odot \text{Mpc}^{-3}$  see Cross et al. 2001).

When selection effects are taken into account properly, the luminosity functions of recent surveys agree very well. The 2dFGRS and SDSS luminosity functions give the same  $M^*$  and  $\phi^*$  values, but disagree on the values of  $\alpha$ . The differences in  $\alpha$  are likely to be due to a combination of input catalogue incompleteness, filter used and different clustering corrections.

§5 suggests that if a deep isophote is used and total magnitudes are recovered, then traditional magnitude dependent estimators can be used. However, future work at higher redshifts or low redshift work on extremely faint galaxies will include many galaxies close to the limits of the detection threshold, where surface brightness dependencies are strong. The BBD provides a framework that allows one to determine whether a bias is present (see Fig. 3) and to correct for it. In addition, it produces parameters such as  $\beta_\mu$  and  $\sigma_\mu$  which could be used to place constraints on galaxy formation and evolution models.

Our conclusion is that after a quarter of a century we need to upgrade our representation of the space-density of galaxies to now include surface brightness. The BBD and BBF provide an excellent starting point and should lead to more reliable and consistent measurements of the local luminosity density as well as providing new constraints on galaxy formation models.

**Table 1.** Bivariate Brightness Function Parameters

Parameter	2dFGRS	dJ & L (2000)
$M_{b_j}^* - 5 \log h / \text{mag}$	$-19.72 \pm 0.08$	$-19.67 \pm 0.17$
$\alpha$	$-1.05 \pm 0.04$	$-0.93 \pm 0.10$
$\beta_\mu$	$0.281 \pm 0.014$	$0.494 \pm 0.04$
$\mu_{e,b_j}^* / \text{mag arcsec}^{-2}$	$21.90 \pm 0.02$ ( $22.45 \pm 0.02$ )	$22.82 \pm 0.19$
$\sigma_\mu$	$0.517 \pm 0.012$	$0.61 \pm 0.04$

## REFERENCES

- Andreon S., Cuillandre J.-C., Pelló 2000 in IAP Meeting “Constructing the Universe with Clusters of Galaxies”, Eds Durret F. & Gerbal D.
- Bertin E., Arnouts S. 1996, A&AS, 117,393
- Blanton M.R. et al. 2001, AJ, 121, 2358
- Cholóniewski J. 1985, MNRAS, 214, 197
- Coleman G.D., Wu C.C., Weedman D.W. 1980, ApJS, 43,393
- Cross, N. et al. 2001, MNRAS, 524, 825
- Dalcanton J.J., Spergel D.N., Summers F.J. 1997, ApJ, 482, 659
- de Jong R. & Lacey C. 1999, Ap&SS, 269, 569
- de Jong R. & Lacey C. 1999, in Low Surface Brightness Universe, ASP Conference Series 170, Eds Davies J.I., Impey C. & Phillipps S., p52
- de Jong R. & Lacey C. 2000, ApJ, 545, 781
- Disney M. 1976, Nature, 263, 573
- Driver S.P., Phillipps S., Davies J.I., Morgan I., Disney M.J. 1994, MNRAS, 266, 155
- Driver S. 1999, ApJL, 526, 69
- Efstathiou, G., Ellis, R., Peterson, B. 1988, MNRAS, 232, 431
- Fall S.M., Efstathiou G. 1980, MNRAS, 193, 189
- Ferguson H.C., Binggeli B. 1994, A&AR, 6, 67
- Fukugita M., Shimasaku K. Ichikawa T. 1995, PASP, 107, 945
- Lin, H., Kirshner, R., Shectman, S., Landy, S., Oemler, A., Tucker, D., Schechter, P. 1996, ApJ, 464, 60
- Loveday J., Peterson B.A., Efstathiou, G., Maddox S.J. 1992, ApJ, 390, 338
- Madgwick D.S. Lahav O. + 2dFGRS Team 2001, submitted to MNRAS.
- Maddox S.J., Efstathiou G. Sutherland W.J. 1990, MNRAS, 246, 433
- Marzke, R., Huchra, J., Gellar, M. 1994, ApJ, 428, 43
- Marzke, R., Da Costa, N., Pelligrini, P., Willmer, C., Gellar, M. 1998, ApJ, 503, 617
- Minchin R. 1999, PASA, 16, 12
- Phillipps S., Davies, J.I., Disney M. 1990, MNRAS, 242, 235
- Press, W., Teukolsky, S., Vetterling, W., Flannery, B. 1986 Numerical Recipes in Fortran, The Art of Scientific Computing 2nd Edition.
- Schechter 1976, ApJ, 203, 297
- Sodré, L.(Jr), Lahav, O. 1993, MNRAS, 260,285
- van der Kruit P.C. 1987, A&A, 173, 59
- Yasuda N., et al. 2001, submitted to AJ, astro-ph/0105545
- Zucca, E. et al. 1997, A&A, 326, 477

## APPENDIX A: TESTING THE FACE-ON PARAMETERS

In Appendix A of Cross et al. (2001), we calculated the difference between our corrected magnitudes and total mag-



**Table 2.** Table of Schechter parameters for Figure 5

$\mu_{lim}$	Magnitude	$M_{b_j}^* - 5 \log h$	$\phi^*/h^3 \text{Mpc}^{-3}$	$\alpha$	$j_{b_j}/10^8 h L_\odot \text{Mpc}^{-3}$
26.0	Isophotal	$-19.54 \pm 0.02$	$(1.95 \pm 0.04) \times 10^{-2}$	$-1.057 \pm 0.01$	$1.74 \pm 0.05$
25.0	Isophotal	$-19.36 \pm 0.02$	$(1.85 \pm 0.04) \times 10^{-2}$	$-1.058 \pm 0.01$	$1.40 \pm 0.05$
24.0	Isophotal	$-19.10 \pm 0.02$	$(1.52 \pm 0.04) \times 10^{-2}$	$-1.090 \pm 0.01$	$0.928 \pm 0.05$
23.5	Isophotal	$-18.86 \pm 0.02$	$(1.36 \pm 0.04) \times 10^{-2}$	$-1.083 \pm 0.01$	$0.662 \pm 0.05$
23.0	Isophotal	$-18.66 \pm 0.02$	$(1.00 \pm 0.04) \times 10^{-2}$	$-1.121 \pm 0.01$	$0.417 \pm 0.05$
26.0	Corrected	$-19.64 \pm 0.02$	$(2.00 \pm 0.04) \times 10^{-2}$	$-1.056 \pm 0.01$	$1.96 \pm 0.05$
25.0	Corrected	$-19.54 \pm 0.02$	$(1.96 \pm 0.04) \times 10^{-2}$	$-1.055 \pm 0.01$	$1.75 \pm 0.05$
24.0	Corrected	$-19.34 \pm 0.02$	$(1.83 \pm 0.04) \times 10^{-2}$	$-1.051 \pm 0.01$	$1.36 \pm 0.05$
23.5	Corrected	$-19.22 \pm 0.02$	$(1.62 \pm 0.04) \times 10^{-2}$	$-1.062 \pm 0.01$	$1.08 \pm 0.05$
23.0	Corrected	$-19.00 \pm 0.02$	$(1.43 \pm 0.04) \times 10^{-2}$	$-1.021 \pm 0.01$	$0.760 \pm 0.05$
26.0	Total	$-19.74 \pm 0.02$	$(2.00 \pm 0.04) \times 10^{-2}$	$-1.066 \pm 0.01$	$2.16 \pm 0.05$
25.0	Total	$-19.74 \pm 0.02$	$(2.00 \pm 0.04) \times 10^{-2}$	$-1.065 \pm 0.01$	$2.16 \pm 0.05$
24.0	Total	$-19.72 \pm 0.02$	$(2.00 \pm 0.04) \times 10^{-2}$	$-1.058 \pm 0.01$	$2.11 \pm 0.05$
23.5	Total	$-19.66 \pm 0.02$	$(1.81 \pm 0.04) \times 10^{-2}$	$-1.037 \pm 0.01$	$1.78 \pm 0.05$
23.0	Total	$-19.62 \pm 0.02$	$(1.08 \pm 0.04) \times 10^{-2}$	$-0.975 \pm 0.01$	$0.989 \pm 0.05$

nitudes compared to isophotal magnitudes and total magnitudes for 8 galaxy types. We have extended this by taking seeing into account and looking at the errors in effective surface brightness too. However, the galaxies are still face on. Fig A1 shows the errors in magnitudes for isophotal magnitudes (triangles) and our corrected magnitudes (squares) compared to total magnitudes, for the 2dFGRS data —  $\mu_{lim} = 24.67$  and seeing of  $2''$ . Each is shown for 3 different redshifts, 0.05, 0.1 and 0.15. The corrected magnitudes are quite good with a typical offset of 0.1 mag and the greatest offset is 0.2 mag for an elliptical galaxy with a redshift of 0.1 or 0.15 and a  $z = 0.15$  LSBG, with a bulge-to-ratio of 0.12. The isophotal magnitudes are much worse, with a typical offset of 0.4 mag and the greatest offset is 2.1 mag for the  $z=0.15$  LSBG. There is only a weak redshift dependence for the corrected magnitudes.

Fig A2 shows the bias in effective surface brightness for our correction, before and after seeing. There is up to  $1.75 \text{ mag arcsec}^{-2}$  offset before seeing is taken into account and up to  $0.7 \text{ mag arcsec}^{-2}$  after seeing is taken into account. The greatest offsets occur in galaxies with large bulges, such as Sa/S0s and ellipticals. The average offset will be around  $0.4 \text{ mag arcsec}^{-2}$ .

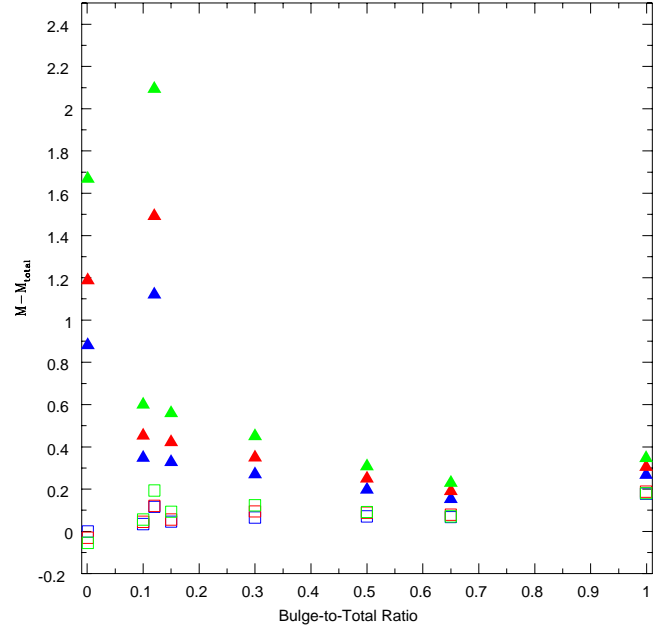
However, there is a significant bias in the effective surface brightnesses. Ellipticals have their effective surface brightnesses underestimated by  $\sim 0.5 \text{ mag arcsec}^{-2}$ ; S0 and Sa galaxies have their effective surface brightnesses overestimated by up to  $0.5 \text{ mag arcsec}^{-2}$ . Late type spirals and irregulars will only have a negligible effect.

However late types will be more effected by inclination. To model this, we have assumed that a disk galaxy is optically thin and has no internal extinction.

A galaxy of area  $A_{iso}$ , was assumed to have an isophotal radius  $r_{iso}$ , given by:

$$A_{iso} = \pi r_{iso}^2 \quad (\text{A1})$$

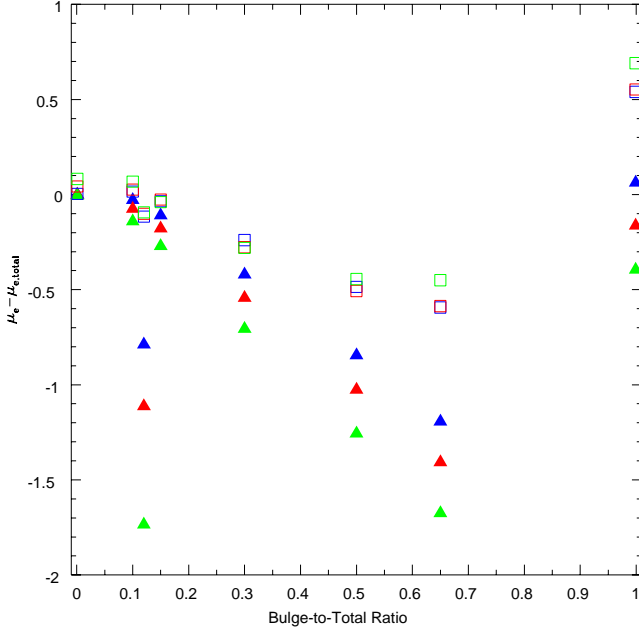
The isophotal magnitude and radius were used to calculate the total magnitude and the effective surface brightness  $\mu_e$  as described in Cross et al. 2001.



**Figure A1.** This plot shows the bias in corrected magnitudes and isophotal magnitudes as a function of bulge-to-total ratio for three different redshifts,  $z = 0.05, 0.10, 0.15$ . The squares show the offset between exponentially corrected magnitudes (Cross et al. 2001) and total magnitudes for an elliptical galaxy,  $B/T = 0.999$ , an S0 galaxy,  $B/T = 0.65$ , Sa-Sd galaxies,  $B/T = 0.5 - 0.1$ , an irregular galaxy,  $B/T = 0.001$ , and a LSBG with,  $\mu_o = 23 \text{ mag arcsec}^{-2}$  and  $B/T = 0.12$ . The offset is low generally around 0.1 mag, with a weak dependence on redshift. The triangles show the difference between isophotal magnitudes and total magnitudes as a function of bulge-to-total ratio for the same galaxies. There is a much greater difference, which is a stronger function of redshift.

However a disk galaxy, inclined at an angle,  $i$  with major axis  $a$  and minor axis  $b$  has an area

$$A_{iso} = \pi ab \quad (\text{A2})$$



**Figure A2.** This plot shows the bias in effective surface brightness for exponentially corrected magnitudes as a function of bulge-to-total ratio. The galaxies are the same type as described in Fig. A1. The triangles represent the offset when seeing is not taken into account, and the squares show the effect of taking seeing into account. There is a weak dependence on redshift.

where

$$b = a \cos(i) \quad (\text{A3})$$

The surface brightness has increased at each point by a factor  $\frac{1}{\cos(i)}$ , increasing the semi-major axis until  $\mu(a) = \mu_{lim}$ .

$$a = \alpha 0.4 \ln(10) [\mu_{lim} - \mu_0 - 2.5 \log_{10} \cos(i)] \quad (\text{A4})$$

where  $\alpha$  is the disk scale length. This increases the isophotal flux of the galaxy, as well as the isophotal radius.

$$r_{iso} = \sqrt{ab} \quad (\text{A5})$$

$$m_{iso} = m_{tot} - 2.5 \log_{10} f \quad (\text{A6})$$

$$f = 1 - \left(1 + \frac{a}{\alpha}\right) e^{-\frac{a}{\alpha}}$$

An exponential profile is fitted to these parameters as in Cross et al. (2001). The central surface brightness and total magnitude are calculated for this galaxy assuming that the galaxy is face on. The error in the central surface brightness is the difference between the true central surface brightness and the calculated central surface brightness. The error in the effective surface brightness is exactly the same, as the difference between central and effective surface brightness is a constant for an exponential profile. The total magnitude calculated above is the same as the true total magnitude.

$$\Delta \mu_0 = \mu_{0, meas} - \mu_{0, true} \quad (\text{A7})$$

The probability of a galaxy of inclination  $\theta$  lying between  $i$  and  $i + di$  is:

$$P(i < \theta \leq i + di) = \frac{1}{2} \sin(i) di \quad (\text{A8})$$

Therefore the mean difference in measured and face-on effective surface brightness is:

$$\Delta \bar{\mu}_e = \frac{1}{2} \int_0^{\pi/2} \Delta \mu_0 \sin(i) di = -0.477 \quad (\text{A9})$$

Thus the calculated effective surface brightnesses of late-type galaxies may be 0.48 mag too bright. However more complicated effects such as seeing, the thickness of the disk and internal extinction will all tend to reduce the measured surface brightness of edge on disks to a greater degree than face-on disks, reducing the mean offset.

The overall effects of bulge-disk decomposition and inclination appear to be to make  $M^*$  brighter by 0.1 mag and  $\mu_e^*$  fainter by 0.55 mag arcsec $^{-2}$ , 0.5 mag arcsec $^{-2}$  for Sds due to the bulge and 0.1 mag arcsec $^{-2}$  for Sds due to the bulge and 0.5 mag arcsec $^{-2}$  due to inclination. It is difficult to be precise about this as the morphological mix of galaxies in the 2dFGRS is not known.